

第五章 不定积分

1. 单项选择题:

(1) 设 $f(x)$ 的一个原函数为 5^x , 则 $f'(x) = (\quad)$.

A. $\frac{5^x}{\ln^2 5}$

B. $\frac{5^x}{\ln 5}$

C. $5^x \ln 5$

D. $5^x \ln^2 5$

(2) 若 $f(x)$ 的导函数是 $\cos x$, 则函数 $f(x)$ 有一个原函数是 ().

A. $1 + \sin x$

B. $1 - \sin x$

C. $1 + \cos x$

D. $1 - \cos x$

(3) 设函数 $f(x)$ 可导, $g(x)$ 的不定积分存在, k 为任意常数, 则下列关系正确的是 ().

A. $\int kf(x)dx = k \int f(x)dx$;

B. $\int [2f(x) + 3g(x)]dx = 2 \int f(x)dx + 3 \int g(x)dx$;

C. $d \int f(3x)dx = f(3x)$;

D. $\int f'(x)dx = f(x)$.

(4) 设 $F(x)$ 是 $f(x)$ 的一个原函数, 则 $\int \sec^2 x f(\tan x)dx = (\quad)$.

A. $F(\tan x) + C$

B. $F(\sec^2 x) + C$

C. $-F(\tan x) + C$

D. $F(\sec^2 x) + C$

(5) 不定积分 $\int e^{2x}dx = (\quad)$.

A. $e^{2x} + C$

B. $\frac{1}{2}e^{2x} + C$

C. $-e^{2x} + C$

D. $-\frac{1}{2}e^{2x} + C$

(6) 已知 $\int f(x^2)dx = e^{\frac{x}{2}} + C, x > 0$, 则 $f(x) = (\quad)$.

A. $\frac{1}{2}e^{\frac{x}{2}}$

B. $\frac{1}{2}e^{\frac{\sqrt{x}}{2}}$

C. $e^{\frac{x}{2}}$

D. $e^{\frac{\sqrt{x}}{2}}$

(7) 不定积分 $\int x^3 e^{5x^4} dx = (\quad)$.

A. $e^{5x^4} + C$

B. $\frac{1}{20}x^3 e^{5x^4} + C$

C. $\frac{1}{20}e^{5x^4} + C$

D. $20e^{5x^4} + C$

(8) 不定积分 $\int x \cos(x^2 + 1)dx = (\quad)$.

A. $\frac{1}{2}\sin(x^2 + 1) + C$

B. $2\sin(x^2 + 1) + C$

C. $\frac{1}{2}x\sin(x^2 + 1) + C$

D. $2x\sin(x^2 + 1)$

(9)不定积分 $\int xf(x^2)f'(x^2)dx=(\quad)$.

- A. $\frac{1}{4}f^2(x^2)+C$ B. $\frac{1}{2}f^2(x^2)+C$ C. $\frac{1}{4}f(x^2)+C$ D. $4f^2(x^2)+C$

(10)设 $\int f(x)dx = \sqrt{2x^2+1}+C$, 则 $\int xf(2x^2+1)dx=(\quad)$.

- A. $x\sqrt{2x^2+1}+C$ B. $\frac{1}{2}\sqrt{2x^2+1}+C$
C. $\frac{1}{4}\sqrt{2x^2+1}+C$ D. $\frac{1}{4}\sqrt{2(2x^2+1)^2+1}+C$

(11)设 $\int f(x)dx = \frac{1}{2}\ln(1+x^2)+C$, 则 $\int \frac{1}{x}f(x)dx=(\quad)$.

- A. $\arctan x+C$ B. $\operatorname{arccot} x+C$;
C. $\frac{1}{2x}\ln(1+x^2)+C$ D. $-\frac{1}{x}+C$

(12) $\int \arcsin x dx=(\quad)$.

- A. $\arcsin x + \sqrt{1-x^2}+C$ B. $x \arcsin x + \sqrt{1-x^2}+C$
C. $\arcsin x - \sqrt{1-x^2}+C$ D. $x \arcsin x - \sqrt{1-x^2}+C$

(13)不定积分 $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx=(\quad)$.

- A. $\frac{1}{2}\arctan(\tan^2 x)+C$ B. $-\frac{1}{2}\arctan(\tan^2 x)+C$
C. $-\frac{1}{2}x^2+C$ D. $\frac{1}{2}\ln\left|\frac{\sin 2x-1}{\sin 2x+1}\right|+C$

(14)已知 $\int 3x^2 f'(x^3+1)dx = (x^3+1)^2+C$, 则 $f(x)=(\quad)$.

- A. $2x+C$ B. x^2+C C. $(x^3+1)^2+C$ D. x^3+C

2. 填空题:

(1)已知 $\int f(x)dx = \log_2 x + \operatorname{arccot} x + C$, 则 $f(x)=$ _____.

(2)已知 $\int f(x)dx = \arctan \frac{1}{x} + C$, 则 $f'(x)=$ _____.

(3)不定积分 $\int (\frac{1}{x} + \frac{2}{x^2})dx =$ _____.

(4)不定积分 $\int (\frac{1}{\sqrt[3]{x}})dx =$ _____.

(5)不定积分 $\int (\cos 2x - \sin 2x)dx =$ _____.

(6)不定积分 $\int (e^{2x} - \frac{1}{x})dx = \underline{\hspace{2cm}}$.

(7)不定积分 $\int (x^2 - 2^x - \frac{2}{x})dx = \underline{\hspace{2cm}}$.

(8)不定积分 $\int \frac{(1+y)^2}{\sqrt[3]{y}} dy = \underline{\hspace{2cm}}$.

(9)不定积分 $\int (\frac{1}{x} + \frac{2}{x^2})dx = \underline{\hspace{2cm}}$.

(10)不定积分 $\int \cos^3 x \sin x dx = \underline{\hspace{2cm}}$.

(11)不定积分 $\int \frac{1}{1+x^2} dx = \underline{\hspace{2cm}}$.

(12)不定积分 $\int \frac{1}{1+\sin x} dx = \underline{\hspace{2cm}}$.

(13)不定积分 $\int \frac{1}{x} dx = \underline{\hspace{2cm}}$.

(14)不定积分 $\int 3x^2 \sqrt{1+x^3} dx = \underline{\hspace{2cm}}$.

(15)不定积分 $\int \frac{\sqrt{1+2\arctan x}}{1+x^2} dx = \underline{\hspace{2cm}}$.

(16)不定积分 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx = \underline{\hspace{2cm}}$.

(17)不定积分 $\int \frac{1}{\sqrt{4+3x} + \sqrt{3x-2}} dx = \underline{\hspace{2cm}}$.

(18)不定积分 $\int \frac{1}{x^3} \sin \frac{1}{x} dx = \underline{\hspace{2cm}}$.

(19)不定积分 $\int \frac{1}{x^2-4} dx = \underline{\hspace{2cm}}$.

(20)不定积分 $\int \frac{x+1}{x^2-2x+5} dx = \underline{\hspace{2cm}}$.

(21)不定积分 $\int \frac{1+2x^2}{x^2(1+x^2)} dx = \underline{\hspace{2cm}}$.

3. 计算题:

(1) $\int \sin x dx$;

(2) $\int \cos x dx$;

(3) $\int \sec^2 x dx$;

(4) $\int \csc^2 x dx$;

(5) $\int \sec x \tan x dx$;

(6) $\int \csc x \cot x dx$;

$$(7) \int \frac{1}{x} dx;$$

$$(9) \int (\sqrt{x}\sqrt{x}\sqrt{x} + e^x) dx;$$

$$(11) \int (3^x + e^x + \sqrt{x}) dx;$$

$$(13) \int \left(\frac{2}{x} + \frac{x}{4}\right)^2 dx;$$

$$(15) \int \left(\frac{1}{x} - \frac{3}{x^2}\right) dx;$$

$$(17) \int \frac{3x^2 - x^2 e^x - 4}{x^2} dx;$$

$$(21) \int \left(x\sqrt{x}\sqrt{x} + \sqrt[3]{x^2} - \frac{x}{\sqrt{x}}\right) dx;$$

$$(23) \int \frac{1 + 2x^2}{x^2(1 + x^2)} dx;$$

$$(25) \int \sin 4x dx;$$

$$(27) \int e^{-x} dx;$$

$$(29) \int (2x - 1)^2 dx;$$

$$(31) \int \frac{1}{x \ln x} dx;$$

$$(33) \int \frac{1}{\sqrt[3]{3-2x}} dx;$$

$$(35) \int \frac{1}{x(1 + \ln^2 x)} dx;$$

$$(37) \int \sin^5 x \cos x dx;$$

$$(39) \int \frac{2x}{\sqrt{1-x^2}} dx;$$

$$(41) \int \frac{x}{7 + 3x^2} dx;$$

$$(43) \int \frac{\tan x}{\cos^2 x} dx;$$

$$(8) \int \frac{2}{x^2} dx;$$

$$(10) \int \left(\sqrt{x}\sqrt{x}\sqrt{x} - \frac{1}{x}\right) dx;$$

$$(12) \int (2^x + e^x + x^2 \sqrt{x}) dx;$$

$$(14) \int \left(\frac{3}{x} + \frac{x}{4}\right)^2 dx;$$

$$(16) \int \left(\frac{1}{\sqrt{x}} - \frac{3}{\sqrt[4]{x}}\right) dx;$$

$$(18) \int \frac{3x^4 + 3x^2 + 1}{x^2 + 1} dx;$$

$$(22) \int \left(x^4 \sqrt{x} + \sqrt[3]{x} + \frac{1}{x}\right) dx;$$

$$(24) \int \frac{1 + 3x^2}{2x^2(1 + x^2)} dx;$$

$$(26) \int \cos 3x dx;$$

$$(28) \int x e^{-x^2} dx;$$

$$(30) \int (x - 4)^2 dx;$$

$$(32) \int \frac{2}{x^2} e^{\frac{1}{x}} dx;$$

$$(34) \int \frac{(\arctan x)^2}{1 + x^2} dx;$$

$$(36) \int \frac{1}{x\sqrt{1 - \ln^2 x}} dx;$$

$$(38) \int \cos^3 x \sin 2x dx;$$

$$(40) \int \frac{1}{1 + 5x^2} dx;$$

$$(42) \int \frac{1}{x^2 + x - 2} dx;$$

$$(44) \int \frac{\cot x}{\sin^2 x} dx;$$

$$(45) \int \frac{e^x}{\sqrt{1-e^{2x}}} dx;$$

$$(47) \int \sin^2 x \cos^3 x dx;$$

$$(49) \int \tan^3 x dx;$$

4. 求下列不定积分:

$$(1) \int \frac{1}{(4-x)\sqrt{3-x}} dx;$$

$$(3) \int \frac{\sqrt{x}}{1+\sqrt{x}} dx;$$

$$(5) \int \sqrt{e^x-1} dx;$$

$$(7) \int \sqrt{9-x^2} dx;$$

$$(9) \int \frac{1}{x^2 \sqrt{x^2-1}} dx;$$

$$(11) \int \frac{1}{\sqrt{(1+x^2)^3}} dx;$$

$$(13) \int \frac{1}{\sqrt{x^2+a^2}} dx;$$

$$(15) \int \frac{x}{\sqrt{2+4x}} dx;$$

$$(17) \int \frac{1}{x\sqrt{1-x^2}} dx;$$

$$(19) \int \frac{\sqrt{1+\ln x}}{x \ln x} dx;$$

$$(21) \int x e^{-x} dx;$$

$$(23) \int x \sin x dx;$$

$$(25) \int x e^{3x} dx;$$

$$(46) \int \frac{x^3}{\sqrt{4-x^4}} dx;$$

$$(48) \int \sin^5 x \cos x dx;$$

$$(50) \int \cot^3 x dx.$$

$$(2) \int \frac{3x+2}{\sqrt{9-x^2}} dx;$$

$$(4) \int \frac{1+\sqrt{x}}{\sqrt[3]{x}+\sqrt[6]{x}} dx;$$

$$(6) \int \frac{1}{\sqrt[3]{x}+\sqrt{x}} dx;$$

$$(8) \int \frac{1}{x\sqrt{x^2-1}} dx;$$

$$(10) \int x^2 \cos x dx;$$

$$(12) \int \sqrt{a^2-x^2} dx (a>0);$$

$$(14) \int \frac{1}{\sqrt{x^2-a^2}} dx;$$

$$(16) \int \frac{1}{\sqrt{(x^2-9)^3}} dx;$$

$$(18) \int \arctan \frac{1}{x} dx;$$

$$(20) \int \frac{e^{2x}}{\sqrt{3e^x-2}} dx;$$

$$(22) \int \ln(x+1) dx;$$

$$(24) \int x^2 e^x dx;$$

$$(26) \int x \arctan x dx;$$

$$(27) \int x^2 \cos x dx;$$

$$(29) \int \frac{x}{\cos^2 x} dx;$$

$$(31) \int \sec^3 x dx;$$

$$(28) \int (\ln x)^2 dx;$$

$$(30) \int (x-1) \cdot 2^x dx;$$

$$(32) \int x^5 \cos x^2 dx.$$