

第六章 定积分

1. (1) 5; (2) 曲线 $y = \sqrt{1-x^2}$ 与 x 轴围成平面图形的面积, $\frac{\pi}{2}$; (3) 0;
- (4) -1; (5) 0; (6) \geq ; (7) $|x|$; (8) $\frac{1}{10}$;
- (9) $\int_{x^3}^{x^2} e^{t^2+t} dt + x^2(2e^{x^4+x^2} - 3xe^{x^6+x^3})$; (10) $\int_0^x f(t) dt$; (11) 4; (12) 32;
- (13) 递减; (14) 0; (15) 6; (16) $\ln \frac{3}{2}$; (17) $\frac{5}{2}$; (18) $\frac{13}{6}$;
- (19) $-\frac{1}{2}$; (20) $\frac{13}{2}$; (21) $\frac{1}{2}(e-1)$; (22) $\frac{2}{5}$; (23) $4-2\ln 3$; (24) 1;
- (25) $\frac{\pi^3}{96}$; (26) $\pm 5x$; (27) $\frac{1}{2}$; (28) < 0 ; (29) $\frac{\pi}{6}$; (30) $\frac{\pi}{4} + \frac{1}{2}\ln 2$;
- (31) $2e^{-1}$; (32) -1; (33) 2; (34) 收敛; (35) $\frac{\pi^2}{8}$; (36) -1;
- (37) $\frac{3}{2} - \ln 2$; (38) 1; (39) $4\ln 2$.
2. (1) $\frac{1}{8}$; (2) $\frac{1}{\ln 2}$; (3) $\frac{32}{3}$; (4) $e^2 - 1$; (5) $\frac{3}{2} + \ln 2$; (6) 4;
- (7) $\frac{9}{2}$; (8) -2; (9) $\frac{1}{2}$; (10) 1; (11) $\frac{20}{\ln 5}$; (12) 1;
- (13) $\frac{1}{8}$; (14) $\frac{3 \times 2^{\frac{15}{4}} - 61}{21}$; (15) $\frac{8}{3}$; (16) $\frac{26}{15}$; (17) $8 - 4x$;
- (18) 8; (19) $(\arctan \sqrt{2})^2 - \frac{\pi^2}{16}$; (20) $\frac{1}{8}$; (21) $1 + \ln 2 - \ln(1+e)$;
- (22) $1 - \cos 1$; (23) $\frac{85}{66}$; (24) $4\sin 1$; (25) $\frac{\pi}{6}$; (26) $\frac{1}{2}\arctan 2$; (27) $\frac{5}{3}$;
- (28) $1 - \frac{\pi}{4}$; (29) $\sqrt{2} - \frac{2\sqrt{3}}{3}$; (30) $\frac{5\pi^2}{144}$.
3. (1) 0; (2) 1; (3) 2; (4) 2.
4. (1) $\frac{1}{2a^2}$; (2) 1; (3) $\frac{1}{2}$; (4) $\frac{\sqrt{2}\pi}{2}$; (5) $\frac{1}{4}\ln 2$.
5. $a = -1$.
6. $p = 2$.
7. 发散.

$$8. \varphi'(x) = \begin{cases} \frac{xf(x) - \int_0^x f(u)du}{x^2}, & x \neq 0, \\ \frac{A}{2}, & x = 0, \end{cases} \quad \text{则 } \varphi'(x) \text{ 在 } x=0 \text{ 处连续.}$$

$$9. k > 1 \text{ 时收敛于 } \frac{1}{k-1}, \quad k \leq 1 \text{ 时发散.}$$

$$10. 3 - \frac{1}{e} + \frac{\pi}{4}.$$

$$11. \frac{1}{2} - \frac{1}{2e^4} + \tan \frac{1}{2}.$$

$$12. \frac{1}{4}.$$

$$13. 1.$$

$$14. \frac{3}{4}.$$

$$14. 4 \ln 2 - \frac{3}{2}.$$

$$15. \frac{1}{1+x^2} + \frac{\pi}{4-\pi} \sqrt{1-x^2}.$$

$$16. x^2 - \frac{4}{3}x + \frac{2}{3}.$$

$$17. \frac{3x^2 \sin x^3}{e^{2x} + 2y \sin y^2} dx.$$

$$18. y = \frac{4}{5}x.$$

$$19. 1.$$

$$20. 1.$$

$$21. \frac{64}{3}.$$

$$22. 2(\sqrt{2}-1).$$

$$23. \frac{14}{3}\sqrt{7} + \frac{20}{3}.$$

$$24. b = \sqrt{2} - 1, \quad a \text{ 为大于零的任意常数.}$$

$$25. y = \frac{1}{4}x - 1 + \ln 4.$$

$$26. (1) a = \frac{1}{e}, \quad (e^2, 1); \quad (2) S = \frac{1}{6}e^2 - \frac{1}{2}.$$

$$27. S = \frac{1}{2}e^2 + \frac{1}{4e^2} + \frac{1}{2}.$$

$$28. V = \frac{108}{5} \pi.$$

$$29. V = \frac{14}{15} \pi.$$

$$30. V = \pi \left(\frac{1}{2} e^4 + \frac{3}{2} e^2 \right).$$

$$31. V_x = \frac{512}{15} \pi, \quad V_y = 64\pi.$$

$$32. (1) a = \frac{1}{\sqrt{2}}, \quad S \text{ 最小值为 } \frac{2 - \sqrt{2}}{6}; \quad (2) V = \frac{\sqrt{2} + 1}{30} \pi.$$

$$33. (1) y = 2x - 1; \quad (2) S = \frac{1}{12} \quad (3) V = \frac{\pi}{30}.$$

$$34. (1) y = 1; \quad (2) V = (1 + 4\ln 2 - 3\ln 3)\pi.$$