

第七章 微分方程

1. (1) C; (2) D; (3) A; (4) C; (5) B; (6) C;
 (7) A; (8) B; (9) B; (10) D; (11) A; (12) D;
 (13) B; (14) A; (15) D; (16) A; (17) D; (18) C;
 (19) B; (20) A; (21) D; (22) B; (23) B; (24) A;
 (25) C; (26) B; (27) B; (28) B; (29) A; (30) B;
 (31) D; (32) D; (33) C; (34) A; (35) C; (36) C;

2. (1) 4; (2) 2; (3) 3;
 (4) $ydy = \frac{x-1}{x} dx$; (5) $\frac{dy}{y} = -x^2 dx$; (6) $\pm \sqrt{C - \cos x}$;
 (7) $-\ln(x+C)$; (8) $-\ln(\cos x + C)$; (9) $e^{e^{x+C}}$;
 (10) $2x^2$; (11) $-e^{\sqrt{1-x^2}}$; (12) $y' + \frac{y}{\sin x} = 0$;
 (13) $\frac{dy}{dx} + \frac{e^{\sin x}}{x^2} y = 0$; (14) $(e^x + x + C)(e^x - 1)$; (15) $(x-e)\ln x$;
 (16) $2e^{2x^2} - 2$; (17) $e^{-x} + C_1 x + C_2$; (18) $4e^{\frac{x}{2}} - 9\cos\frac{x}{3} + C_1 x + C_2$;
 (19) $C_1(\arcsin x + \sqrt{1-x^2}) + C_2$; (20) $C_2 \sqrt[3]{3x+C_1}$; (21) 是;
 (22) 无关; (23) 不是; (24) 其一个特解;
 (25) $r^2 + pr + q = 0$; (26) $C_1 e^{-x} + C_2 e^x$; (27) $C_1 e^{-2x} + C_2 e^{6x}$;
 (28) 3.

3. (1) $y = e^{Cx^2}$ (C 为任意常数);
 (2) $y = \arctan C(3 - e^x)^2$ (C 为任意常数) 或 $y = k\pi$ ($k = \pm 1, \pm 2, \dots$);
 (3) $y = \arctan C \tan x$ (C 为任意常数) 或 $y = k\pi$ ($k = \pm 1, \pm 2, \dots$);
 (4) $\ln|y| - \frac{y^2}{2} = 3x - 3\arctan x + C$ (C 为任意常数) 或 $y = 0$;
 (5) $y^2 + e^{-y} = 2x^2 + e^x + C$ (C 为任意数); (6) $ye^y = \frac{C}{x^2 e^x}$ (C 为任意常数);
 (7) $|\cos y| = |\cos x|$ 或 $y = x \pm \pi$; (8) $y = \frac{1}{\ln(1-x^2)+1}$;
 (9) $y = (x \ln x - x)^2$; (10) $y = \cot^2 x$ 或 $y = (2 - \cot x)^2$;
 (11) $y = Cx^{-1} - \frac{\cos x}{x}$ (C 为任意常数); (12) $y = x^3(x-1)e^x + Cx^3$ (C 为任意常数);
 (13) $y = -(x^2 + 1) + Ce^{x^2}$ (C 为任意常数); (14) $y = x \ln \ln x + Cx$ (C 为任意常数);
 (15) $y = \frac{1}{3}xe^{2x} + Cxe^{-x}$ (C 为任意常数); (16) $y = 3x + C \sec x$ (C 为任意常数);
 (17) $y = (x+2)e^{\arctan^2 x}$; (18) $y = x^3 - 3x^2 \ln|x| + x^2$;

$$\begin{aligned}
(19) y &= \frac{(x-1)e^x}{x}; & (20) y &= (1-x)(1+x^2); \\
(21) y &= \frac{1}{8}x^4 + \frac{1}{16}\sin 2x + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4 \quad (C_1, C_2, C_3, C_4 \text{ 为任意常数}); \\
(22) y &= 4(x+1)e^{-2x} + C_1x + C_2 \quad (C_1, C_2 \text{ 为任意常数}); \\
(23) y &= \frac{x^2}{2} + \frac{x^4}{12} + C_1x + C_2 \quad (C_1, C_2 \text{ 为任意常数}); \\
(24) y &= C_1(e^x + x) + C_2 \quad (C_1, C_2 \text{ 为任意常数}); \\
(25) y &= \frac{1}{2}e^{2x} + C_1e^x + C_2 \quad (C_1, C_2 \text{ 为任意常数}); \\
(26) y &= -3\cos x + C_1\sin x + C_2 \quad (C_1, C_2 \text{ 为任意常数}); \\
(27) y &= \frac{1}{2}(1 - e^{-x^2}); & (28) y &= e^{-x^2} - 1; \\
(29) y &= -\frac{2}{3}x^3 - x; & (30) y &= e^{-\sin x} - 1; \\
(31) y &= \frac{C_1}{4}(C_2 \pm x)^2 + \frac{1}{C_1} \quad (C_1, C_2 \text{ 为任意常数}); \\
(32) y &= \pm \sqrt{C_1(x+C_2)^2 - \frac{1}{C_1}} \text{ 或 } y = \sqrt{C_2 \pm 2x} \quad (\text{其中 } C_2 \text{ 为任意常数, } C_1 \neq 0); \\
(33) y &= -\ln \frac{e^{C_3 - C_2x} - 1}{C_2} \quad (\text{其中 } C_3 \text{ 为任意常数, } C_2 \neq 0) \text{ 或 } y = -\ln(C_3 - x) \text{ 或 } y = C_1; \\
(34) y &= \frac{1}{3}\ln(C_1x + C_2) \quad (C_1, C_2 \text{ 为任意常数}) \text{ 或 } y = C_1; \\
(35) y &= \arcsin(C_2x + C_3) \quad (C_2, C_3 \text{ 为任意常数}) \text{ 或 } y = C_1; \\
(36) x &= -(e^y + \frac{y^2}{2} + C_2y) + C_3 \quad (\text{其中 } C_2, C_3 \text{ 为任意常数}) \text{ 或 } y = C_1; \\
(37) x &= -\sin y + C_2y + C_3 \quad (\text{其中 } C_2, C_3 \text{ 为任意常数}) \text{ 或 } y = C_1; \\
(38) y &= \frac{1}{64}x^4; & (39) y &= 4(x-1)^{-2} \text{ 或 } y = 4(x-5)^{-2}; \\
(40) y &= C_1e^{-\sqrt{2}x} + C_2e^{\sqrt{2}x} \quad (C_1, C_2 \text{ 为任意常数}); & (41) y &= C_1 + C_2e^{2x} \quad (C_1, C_2 \text{ 为任意常数}); \\
(42) y &= C_1e^{-x} + C_2e^{-4x} \quad (C_1, C_2 \text{ 为任意常数}); & (43) y &= (C_1 + C_2x)e^{-5x} \quad (C_1, C_2 \text{ 为任意常数}); \\
(44) y &= C_1e^{(1+\frac{\sqrt{2}}{2})x} + C_2e^{(1-\frac{\sqrt{2}}{2})x} \quad (C_1, C_2 \text{ 为任意常数}); \\
(45) y &= 1 + e^{3x}; & (46) y &= e^x + e^{2x}; \\
(47) y &= C_1e^x + C_2e^{-x} + 2x - 1 \quad (C_1, C_2 \text{ 为任意常数}); \\
(48) y &= C_1e^{2x} + C_2e^{-2x} - \frac{4}{3}e^x \quad (C_1, C_2 \text{ 为任意常数}); \\
(49) y &= (x^2 - x + C_1)e^x + C_2e^{-x} \quad (C_1, C_2 \text{ 为任意常数});
\end{aligned}$$

$$(50) y = C_1 e^{-x} + (C_2 - 2x)e^{-4x} \quad (C_1, C_2 \text{ 为任意常数});$$

$$(51) y = (C_1 + C_2 x + \frac{1}{2} x^2) e^x \quad (C_1, C_2 \text{ 为任意常数});$$

$$(52) y = (C_1 + C_2 x + 2x^3) e^x \quad (C_1, C_2 \text{ 为任意常数});$$

$$(53) y = (C_1 - x - 1) e^x + C_2 e^{2x} + x + 3 \quad (C_1, C_2 \text{ 为任意常数});$$

$$(54) y = (C_1 + C_2 x) e^{-2x} - x + \frac{3}{2} + \frac{3}{2} x^2 \quad (C_1, C_2 \text{ 为任意常数});$$

$$(55) y = -5e^x + \frac{7}{2} e^{2x} + \frac{5}{2};$$

$$(56) y = -4xe^x + e^{9x};$$

$$(57) y = e^{2x} (C_1 \cos 3x + C_2 \sin 3x) + \cos 3x - 3 \sin 3x \quad (C_1, C_2 \text{ 为任意常数});$$

$$(58) y = C_1 \cos 2x + (C_2 - 2x) \sin 2x \quad (C_1, C_2 \text{ 为任意常数});$$

$$(59) y = C_1 \cos x + (C_2 - \frac{1}{2} x) \sin x - 3x \sin 2x - 4 \cos 2x \quad (C_1, C_2 \text{ 为任意常数});$$

$$(60) y = (C_1 - x) \cos \frac{x}{2} + C_2 \sin \frac{x}{2} + e^x (\cos x + 8 \sin x) \quad (C_1, C_2 \text{ 为任意常数});$$

$$(61) y = -\cos x - \frac{1}{3} \sin x + \frac{2}{3} \cos 2x;$$

$$(62) y = \frac{\pi}{4} \cos x - \frac{3}{2} \sin x + \frac{1}{2} x \cos x.$$